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Further Development of the Weibull Distribution Method for Evaluating ALARA Performance

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Further Research in This Paper

- Evaluation of Goodness of Fit
- Expanded information for applying Weibull performance indicators
- Examination of Transient Worker effect
- Investigation into Experienced Worker effect

Goodness of Fit

- Statistical models require goodness of fit evaluation.
- Goodness of fit issues with respect to ALARA application of Weibull performance indicators:
  - Formal evaluation of goodness of fit based on Chi-square test.
  - Examination of plots to determine patterns in lack of fit.

Formal Assessment

- Basis: Chi-square test having null hypothesis that data fit the Weibull distribution.
- Procedure: Separate data into *k* bins.
- Test statistic: Sum over all bins of squares of residuals where residuals are (observed – expected) / $\sqrt{}$(expected).
- p-value: Based on Chi-square statistic with *k-2* degrees of freedom.
- Conclusion: Any site having a p-value smaller than the critical value has statistically significant lack of fit and requires further evaluation.

Weibull Plots for Goodness of Fit

- Purpose:
  - To visually examine patterns of goodness of fit .
  - To uncover dose ranges where lack of fit occurs.
- Plots for complementary information:
  - Customized Weibull probability plot based on survival function.
  - Goodness of fit plots bases on Weibull hazard function.
  - High hazard corresponds to low survival.
- Statistical details summarized in the following table.

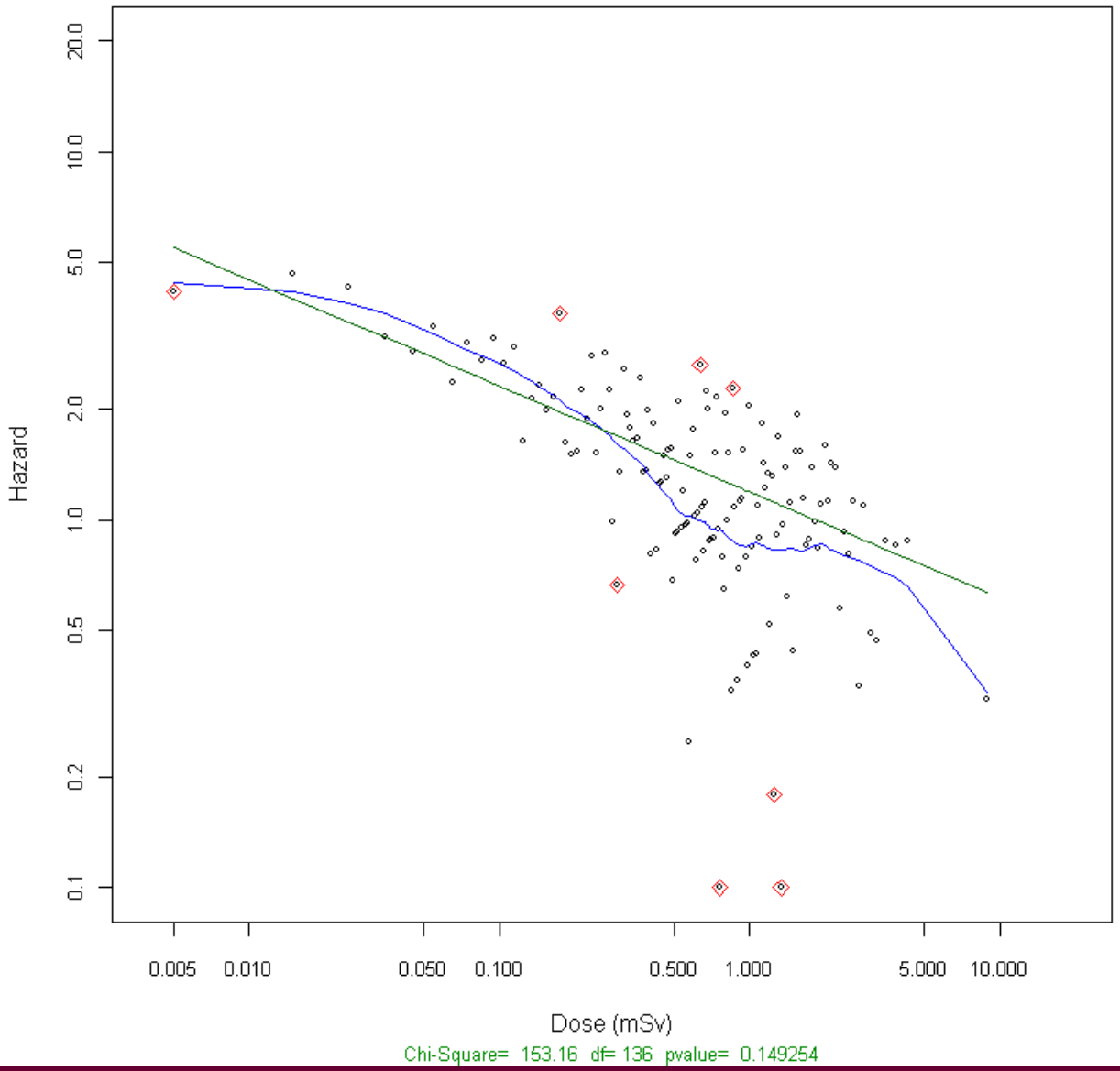
Attribute	Probability Plots	Goodness of Fit Plots
Weibull line		
derived from	S(x) = 1 – F(x): exceedance (also called survival) function; -S(x) represented	h(x): hazard function
line	solid black from upper left to lower right	solid green: upper left to lower right if α□< 1
x-“axis”/ x-axis	$\ln(u_i)$: u_i are ordered distinct values of x; labels adjusted to doses before MIT subtracted; intersects Weibull line at fitted 99th percentile	$\ln(v_i)$: v_i are ordered interval midpoints of x; labels adjusted to match doses with MIT subtracted
y-axis	$\ln(\beta)*\alpha - \alpha* \ln(u_i)$: labels adjusted to values of exceedance function in %	$\{\ln(\alpha)-\alpha \ln(\beta)\}+(\alpha-1)[\ln(v_i)]$: labels adjusted to values of hazard function
Points		
symbol	open circle	open circle
basis	one for each distinct value of x	one for each interval in Chi-squared test
x-coordinate	$\ln(u_i)$	$\ln(v_i)$
y-coordinate	$-\ln(-\ln(S_{emp}(u_i)))$, where $S_{emp}(u_i) = [n/(n+1)]* [1–F_{emp}(u_i)]$, where F_{emp} is the empirical cdf	$\ln(h_{N\text{-}A}(v_i))$, where $h_{N\text{-}A}$ is Nelson-Aalen hazard estimate
Added line		
reference line	dashed green line from upper left to lower right: Weibull line for –slope = 1 and 99% of doses < 1 rem	blue curve: non-parametric smoother that shows change in hazard over dose range

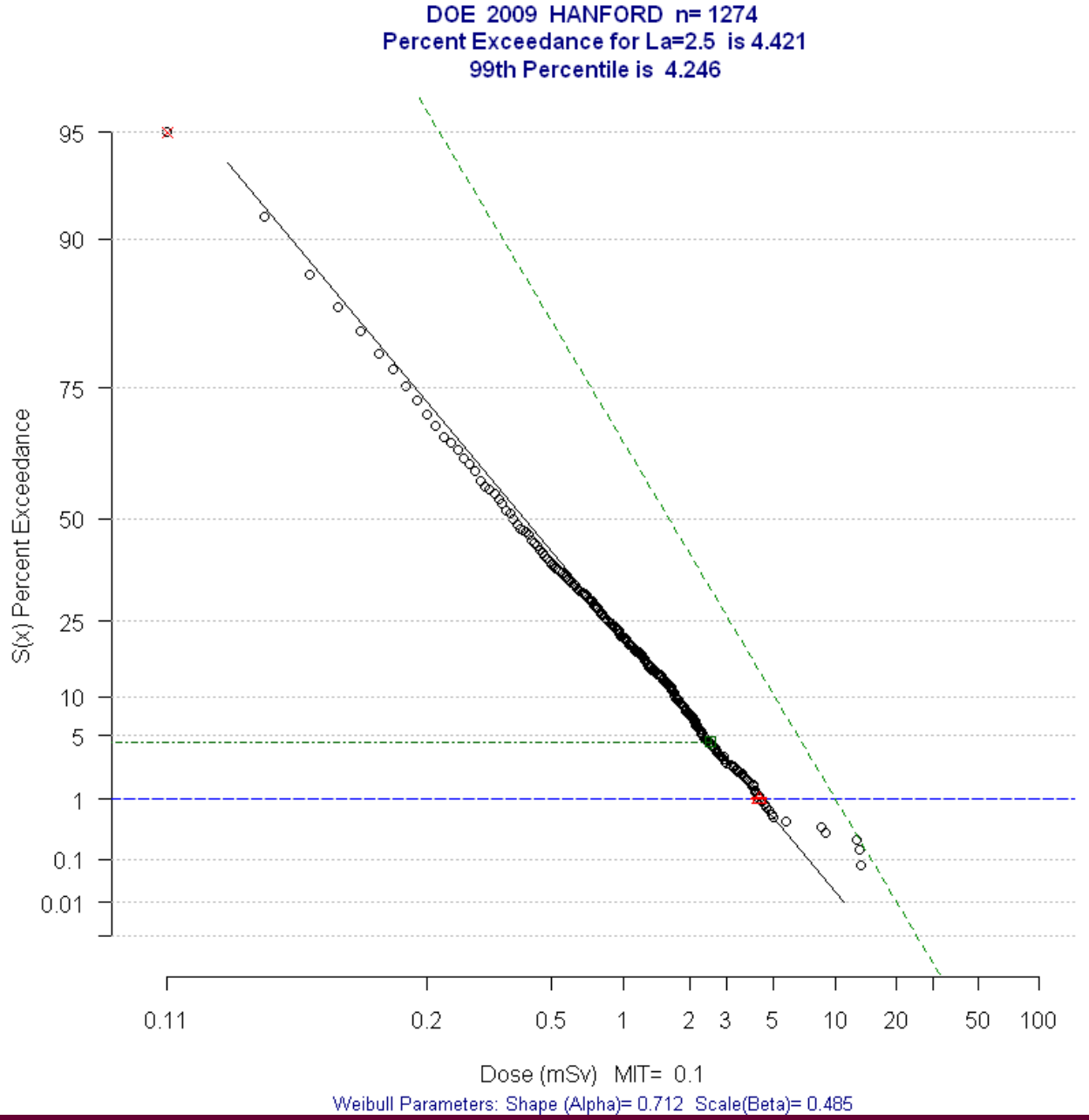
Goodness of Fit Plots

- Method: Examine how close open circles are to green Weibull hazard line.
- Non-critical lack of fit: Low dose region.
- Flag raised: Rising blue curve (non-parametric smoother) covering dose intervals around important values, e.g. 10 mSv.
  - Were administrative criteria applied to pull individuals out of jobs with exposure potential when dose for year approached predetermined value?
- More effective ALARA practices: Actively maintaining each worker's dose as low as reasonably achievable throughout the year.

Example - DOE Hanford 2009

- Information from goodness of fit hazard plot:
  - Points represent intervals used in Chi-square test.
  - p-value = 0.150 so not statistical evidence of lack of fit to Weibull.
  - Non-parametric smoother generally close to hazard line.
- Information from survival probability plot:
  - Points represent unique values in dose distribution.
  - Points generally close to Weibull line except for 6 (out of 1274) at high dose end and one at very low dose.

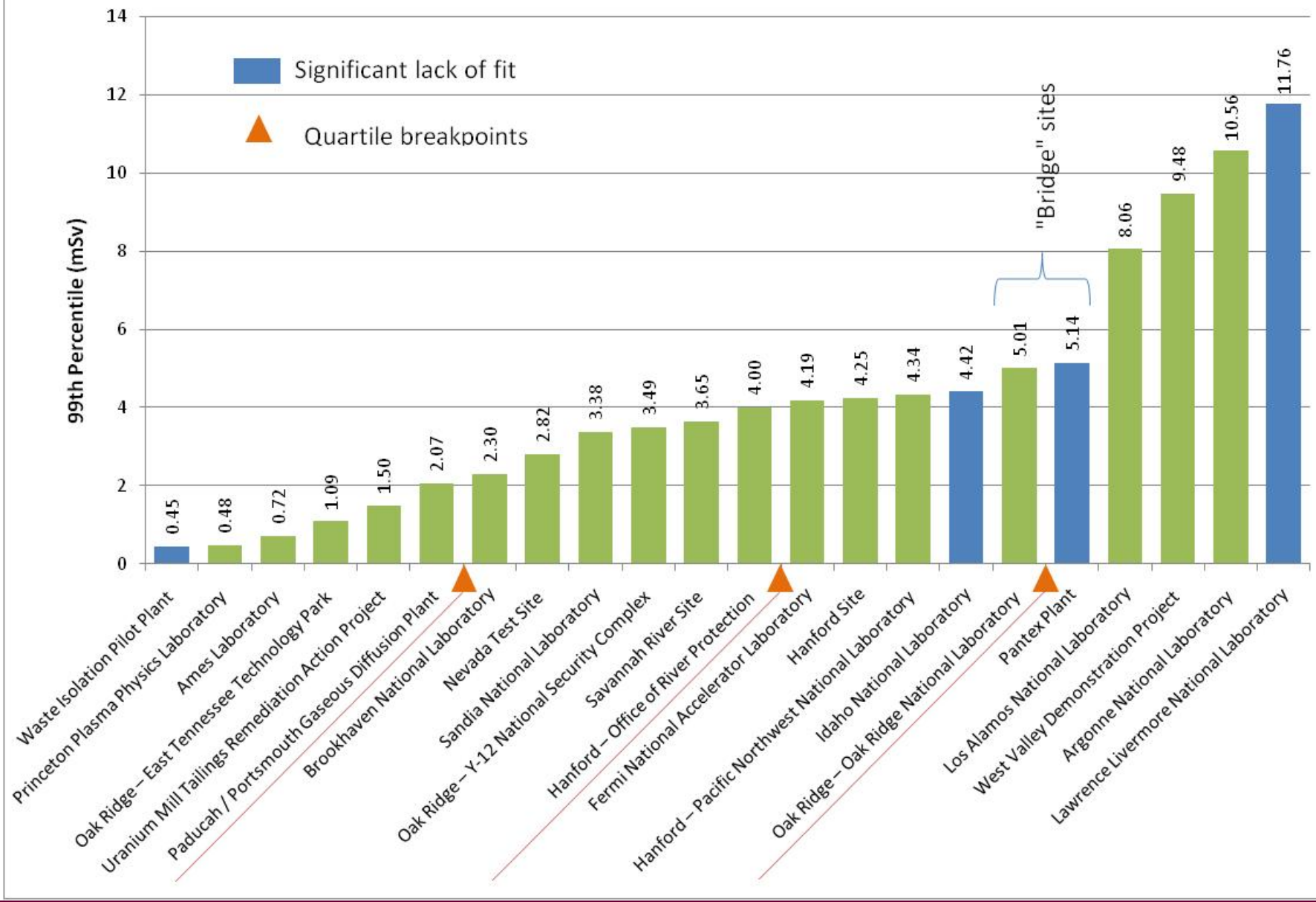

DOE 2009 HANFORD Hazard Function
Hazard
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.005
0.010
0.050
0.100
0.500
1.000
5.000
10.000
Dose (mSv)
Chi-Square= 153.16 df= 136 pvalue= 0.149254


DOE 2009 HANFORD n= 1274
Percent Exceedance for La=2.5 is 4.421
99th Percentile is 4.246
S(x) Percent Exceedance
95
90
75
50
25
10
5
1
0.1
0.01
0.11
0.2
0.5
1
2
3
5
10
20
50
100
Dose (mSv) MIT= 0.1
Weibull Parameters: Shape (Alpha)= 0.712 Scale(Beta)= 0.485

Using Weibull Performance Indicators
Process

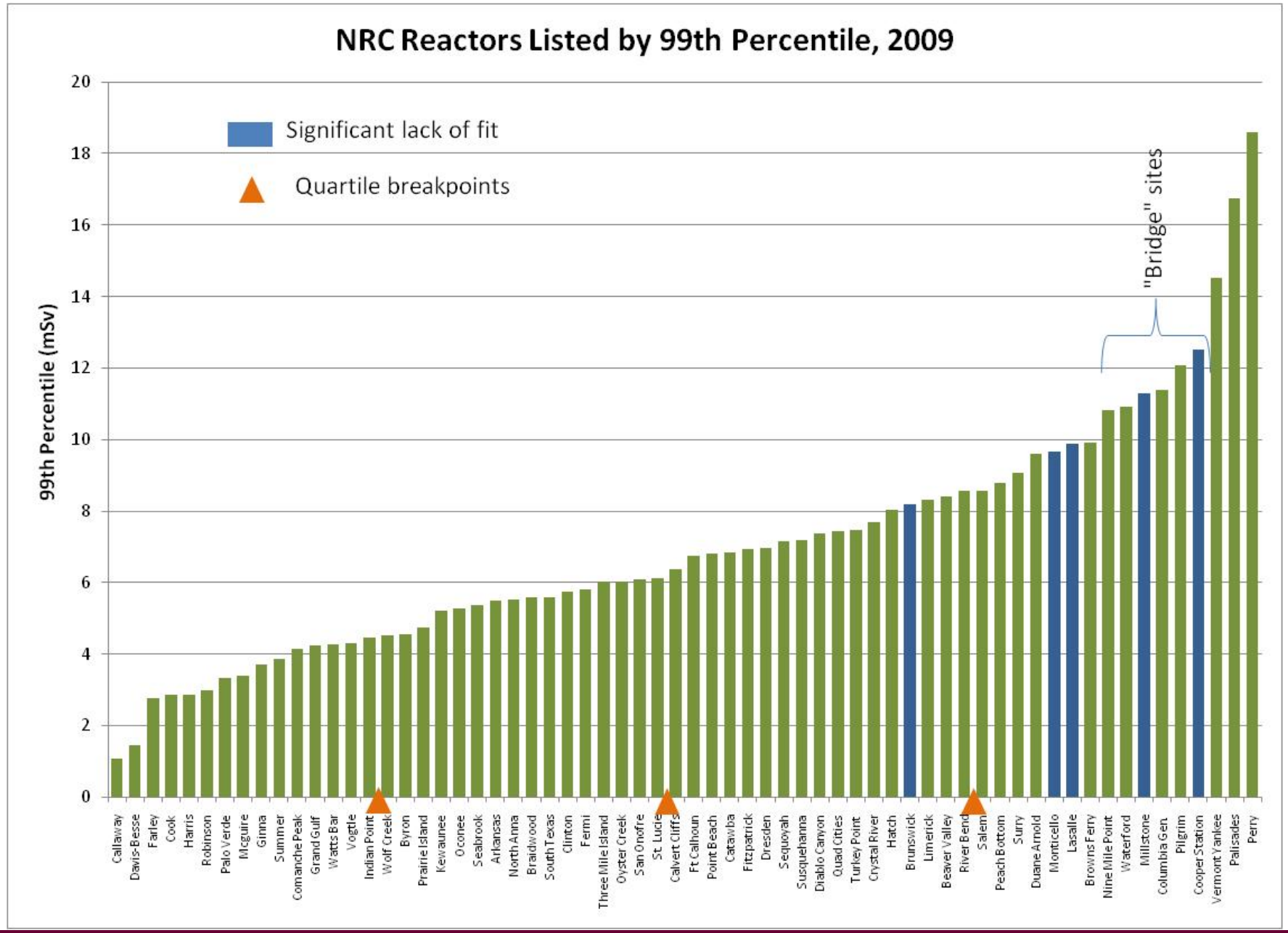
- Check value of shape parameter α: Site not effectively implementing ALARA if α > 1.
  - Exception to rule: α > 1 but very small 99th percentile along with percent exceedance near zero.
- Rank sites by fitted Weibull 99th percentile.
  - Table 2: DOE 2009.
  - Table 3: NRC 2009.
- Examine percent exceedance for additional information.

DOE Sites Listed by 99th Percentile, 2009



Using Weibull Performance Indicators Application to DOE 2009

- Question: Should sites be partitioned into quartiles by 99th percentiles?
- DOE 2009: Gap between first and second sites in highest quartile.
  - Pantex =5.141; LANL= 8.058.
- Alternative to partitioning into quartiles: Use 99th percentiles to identify clusters of sites.
  - Group 1: Sites that do not appear to be implementing ARARA effectively.
    - LANL, West Valley, ANL, and LLNL
  - Group 2: Sites that bridge between Group 1 and remainder of sites that do appear to implement ALARA effectively
    - Pantex, ORNL, and, based on percent exceedance, possibly Fermi lab.


NRC Reactors Listed by 99th Percentile, 2009
99th Percentile (mSv)
0
2
4
6
8
10
12
14
16
18
20
Significant lack of fit
Quartile breakpoints
"Bridge" sites
Callaway
Davis-Besse
Farley
Cook
Harris
Robinson
Palo Verde
Mcguire
Ginna
Summer
Comanche Peak
Grand Gulf
Watts Bar
Vogtle
Indian Point
Wolf Creek
Byron
Prairie Island
Kewaunee
Oconee
Seabrook
Arkansas
North Anna
Braidwood
South Texas
Clinton
Fermi
Three Mile Island
Oyster Creek
San Onofre
St. Lucie
Calvert Cliffs
Ft Calhoun
Point Beach
Catawba
Fitzpatrick
Dresden
Sequoyah
Susquehanna
Diablo Canyon
Quad Cities
Turkey Point
Crystal River
Hatch
Brunswick
Limerick
Beaver Valley
River Bend
Salem
Peach Bottom
Surry
Duane Arnold
Monticello
Lasalle
Browns Ferry
Nine Mile Point
Waterford
Millstone
Columbia Gen
Pilgrim
Cooper Station
Vermont Yankee
Palisades
Perry

Using Weibull Performance Indicators Application to NRC 2009

- Group 1: Vermont Yankee, Palisades, and Perry.
  - High 99th percentiles and UCLs.
  - Palisades and Perry also have over 30% of doses exceeding 3 mSv.
- Group 2: Cooper Station, Pilgrim, Columbia Generating, Millstone, Waterford, and Nine Mile Point.
- Additional sites of interest: Surry and Browns Ferry
  - Percent exceedance above 15%.

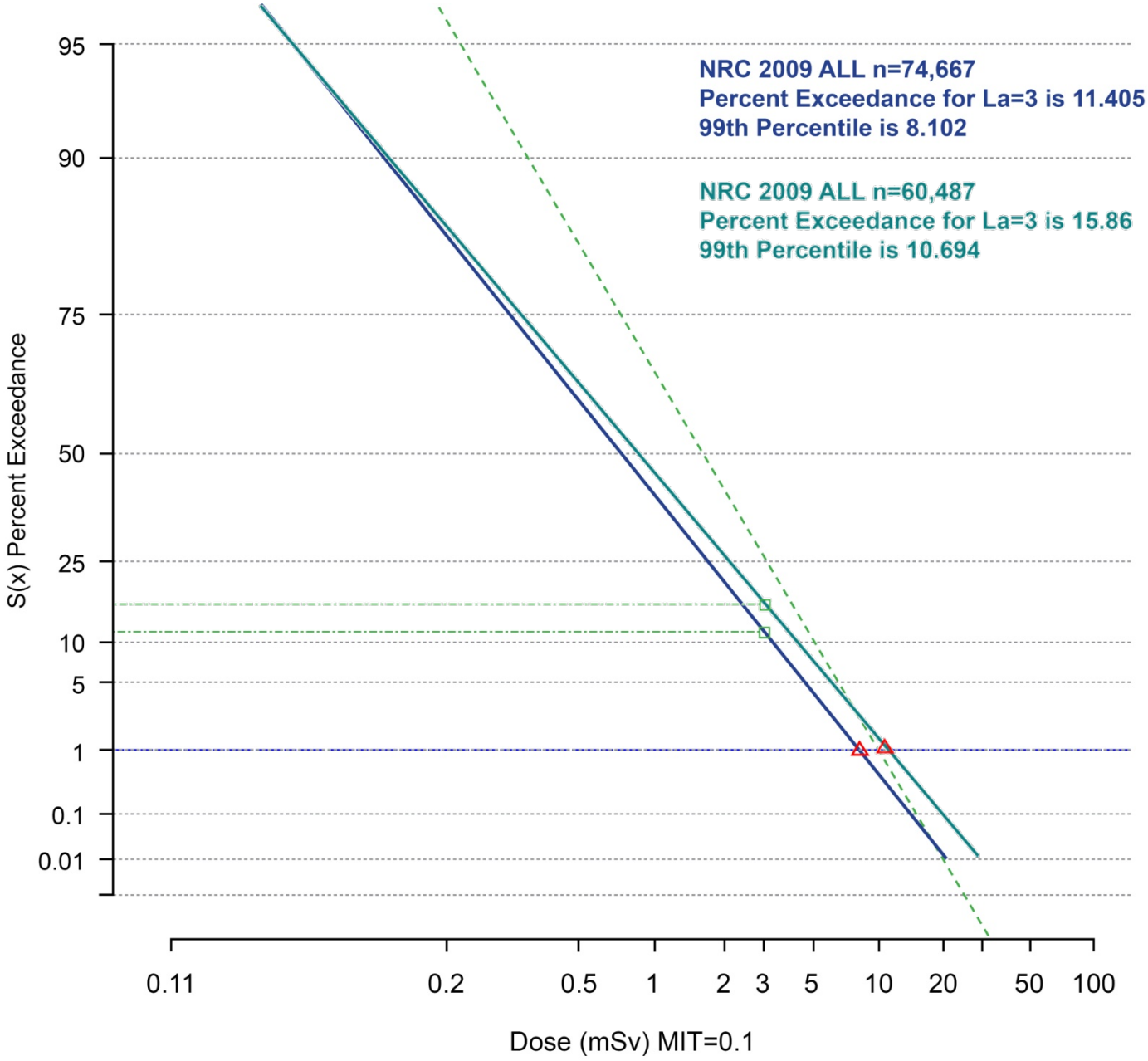
Using the Plots to Assess Weibull Fit

- Sites with statistically significant lack of fit :
  - DOE: INL, LLNL, Pantex, and WIPP.
  - NRC: Brunswick, Cooper Station, LaSalle, Millstone, and Monticello.
- Management must decide whether to use Weibull performance indicators for ALARA evaluation.
- Analysis of Weibull plots – substantial contribution to this decision.
- Detailed discussions of analysis for these sites in Proceedings paper.

Transient Worker Effect

- Transient workers: Monitored at more than one site during the year.
- Doses remain in separate site records for analysis of all NRC sites.
- When adjusted, transient worker doses from separate sites are summed.
- Adjusting for transient workers effects the Weibull results:

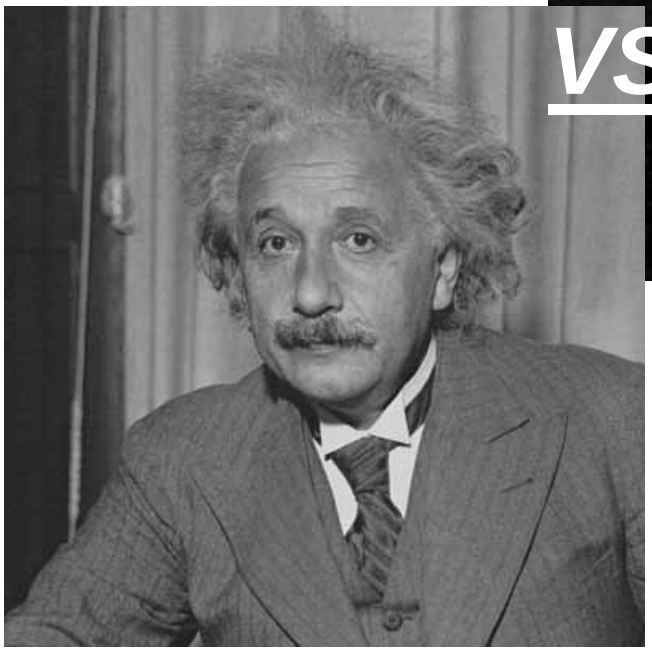
	Dose Separated by Site of Accrual	Dose Combined into One Record
Number of Records	74,667	60,487
Fitted 99th Percentile	8.102 mSv	10.694 mSv
% Exceedance for 3 mSv	11.41%	15.86%
-Slope	0.740	0.707


NRC 2009 ALL n=74,667
Percent Exceedance for La=3 is 11.405
99th Percentile is 8.102
NRC 2009 ALL n=60,487
Percent Exceedance for La=3 is 15.86
99th Percentile is 10.694
S(x) Percent Exceedance
95
90
75
50
25
10
5
1
0.1
0.01
0.11
0.2
0.5
1
2
3
5
10
20
50
100
Dose (mSv) MIT=0.1

Experienced Worker Effect

- Specialists trained to perform job tasks that involve potentially high exposure.
- Using experienced workers among these specialists promotes safety and results in lowest accumulation of collective dose.
- This practice can be an acceptable approach for supporting ALARA.
- Informal sensitivity analysis was carried out using several scenarios:
  - Scenario 1: Crystal River 2009 distribution
  - Scenario 2: The actual distribution is modified by reducing or eliminating the dose received by one individual in the higher dose ranges and distributing this dose among two or three workers in the lower dose ranges.
  - Scenario 3: Same as Scenario 2, with more individuals modified.

One experienced worker



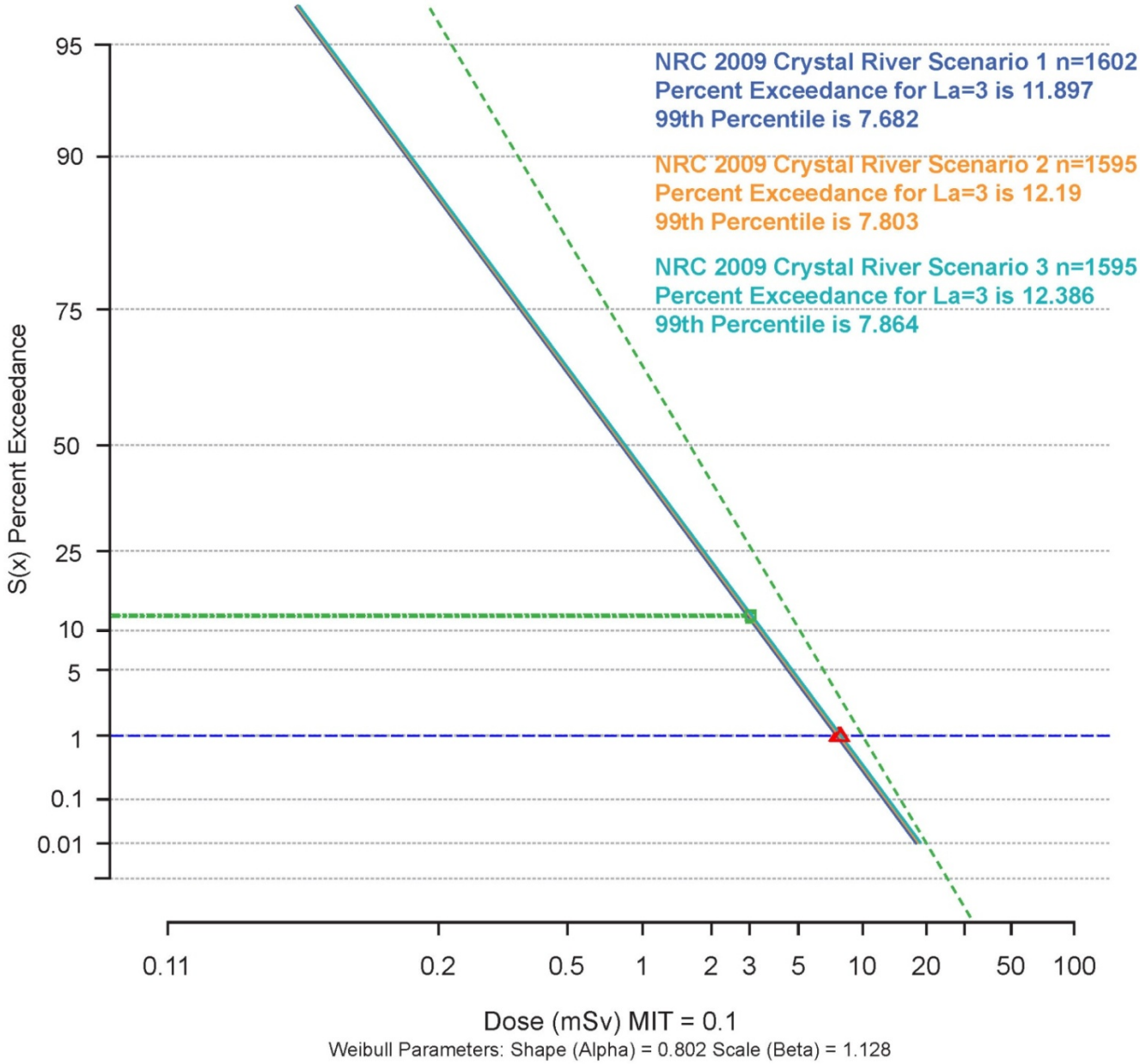
VS

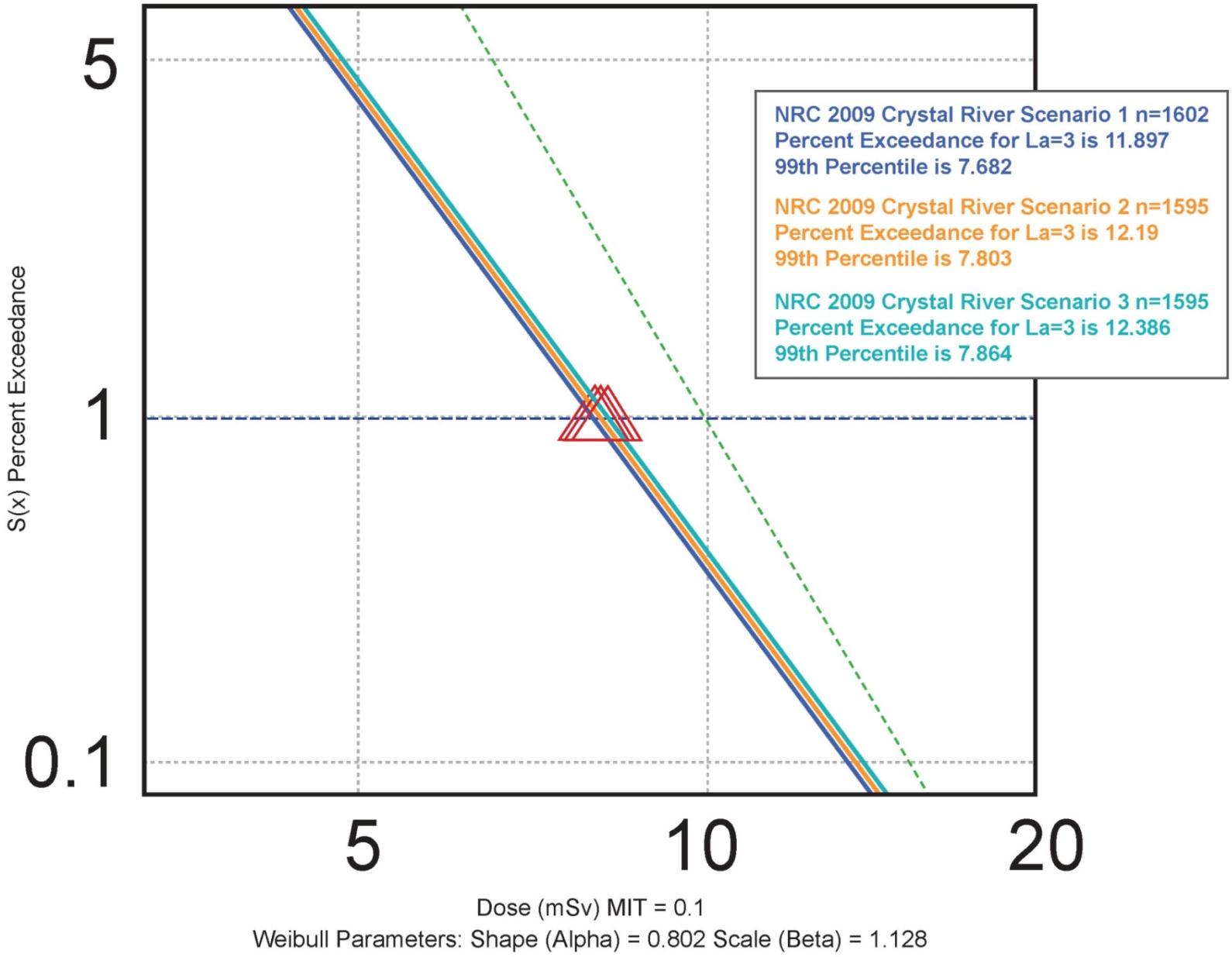
Several ‘less’ experienced workers

Results

- Each successive scenario experienced a small increase in the 99th percentile.
- Effect on 99th percentile of using less experienced workers shows this to be a somewhat less effective ALARA practice.

Scenario	α	β	99th percentile	99th % tile- ucl	% exceedance	nx
1	0.802	1.128	7.682	8.212	11.897	1602
2	0.801	1.143	7.803	8.345	11.148	1595
3	0.802	1.156	7.864	8.410	12.386	1595


NRC 2009 Crystal River Scenario 1 n=1602
Percent Exceedance for La=3 is 11.897
99th Percentile is 7.682
NRC 2009 Crystal River Scenario 2 n=1595
Percent Exceedance for La=3 is 12.19
99th Percentile is 7.803
NRC 2009 Crystal River Scenario 3 n=1595
Percent Exceedance for La=3 is 12.386
99th Percentile is 7.864
S(x) Percent Exceedance
95
90
75
50
25
10
5
1
0.1
0.01
0.11
0.2
0.5
1
2
3
5
10
20
50
100
Dose (mSv) MIT = 0.1
Weibull Parameters: Shape (Alpha) = 0.802 Scale (Beta) = 1.128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Percent Exceedance for La=3 is 12.19
99th Percentile is 7.803
NRC 2009 Crystal River Scenario 3 n=1595
Percent Exceedance for La=3 is 12.386
99th Percentile is 7.864
S(x) Percent Exceedance
5
1
0.1
5
10
20
Dose (mSv) MIT = 0.1
Weibull Parameters: Shape (Alpha) = 0.802 Scale (Beta) = 1.128

Conclusions

- Use maximum likelihood methods to estimate Weibull shape and scale parameters for each site in the group.
- From site-specific parameters calculate fitted 99th percentile for performance indicator and percent exceedance as alternative performance indicator.
- Rank sites by 99th percentile and look for clusters of sites at high end.
- Perform Chi-squared goodness of fit tests to identify sites with statistically significant lack of fit to a Weibull distribution
- Consult Management to determine whether any operational issues affected dose distribution from sites with lack of fit.

Conclusions (continued)

- Use customized probability and goodness of fit plots to investigate patterns of lack of fit.
  - Is lack of fit substantial enough for the Weibull-based performance indicators to be rejected?
  - Are lack of fit contributions to the Chi-squared statistic concentrated in intervals from the very low dose range?
  - Are there values that appear to have been set administratively as an upper bound for an individual's dose?
- Weibull methodology reflected expected impact on ALARA performance indicators in transient and experienced workers analyses.

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